Single-Item Auctions

Setup: - seller has one item to sell
- n bidders, each bidder i has a private valuation v_i

i.e., max willingness to pay

↳ i.e., known only to bidder i

Bidder utility: (1) lose $\Rightarrow$ utility $= 0$
(2) win at price $p \Rightarrow$ utility $= v_i - p$

Ascending (English) auction: winner = last bidder with hand up
selling price = last announced price

Note: winner generally pays less than valuation
(price = 1st one higher that all the other bidder valuations)

Second-Price (Vickrey) Auctions (1961)

(1) each bidder i submits bid b_i [$b_i > v_i$, $b_i < v_i$, $b_i = v_i$ all possible]

(2) winner = highest bidder

(3) selling price = highest bid by someone else (2nd-highest overall)

Key property: Vickrey auction is truthful. (i.e., is a dominant strategy)

(i.e., setting $b_i := v_i$ always maximizes i's utility, no matter what anyone else does)

Thus: strategically trivial for bidders.

Note: very special property! (in fact, unique to Vickrey auctions)

Properties of Second-Price Auctions

Key property: in a second-price auction, truthful bidding is a dominant strategy.

Intuition: auctioneer shades your bid optimally on your behalf.

Bidder utility:

lose $\Rightarrow$ utility $= 0$

win at price p $\Rightarrow$ utility $= v_i - p$

[v_i = bidder i's valuation]

Proof: Fix bidder i, valuation v_i, all bids other than b_i. Let $B = \max_{j \neq i} b_j$.

Note: i's payoff is either 0 (if $b_i \leq B$) or $v_i - B$ (if $b_i \geq B$)

Case 1 [$v_i \leq B$]: max-possible utility $= 0$, achieved by setting $b_i = v_i$

Case 2 [$v_i \geq B$]: max-possible utility $= v_i - B$, achieved by setting $b_i = v_i$

Sponsored Search Auctions

Bidders: advertisers who bid on the keywords you searched for.

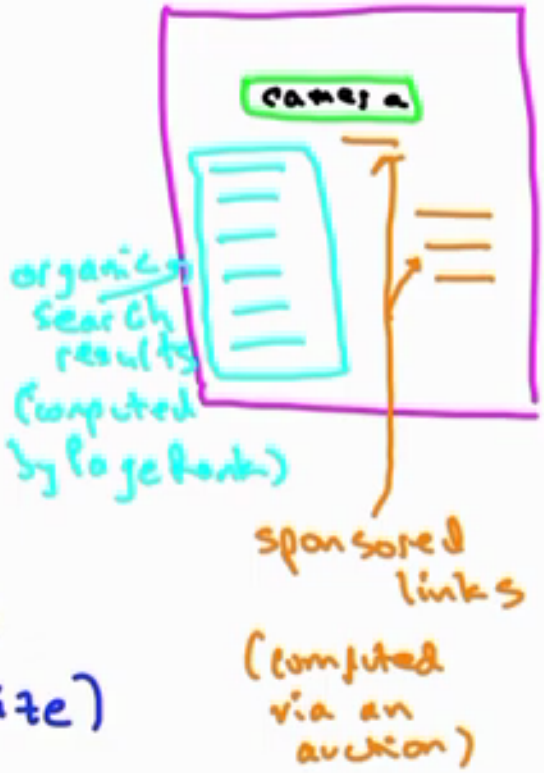
Goods: "slots" on the results page.
- note not identical / interchangeable (higher is better)

Click-through rate (CTR): α_j = probability j^{th} slot gets click on

Assumptions: ① $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$ (k = # of slots)

② α_j independent of slot occupant (easy to generalize)

③ bidder i has valuation v_i per click
⇒ has valuation $\alpha_j v_i$ for an impression in slot j



Generalized Second-Price Auctions

Question: Analog of Vickrey auction for sponsored search?

Step 1: collect bid (per-click) b_i from each advertiser i.

Step 2: decide on the winners.

- assign j^{th} highest bidder to j^{th} best slot [for $j=1,2,\ldots,k$]

Step 3: decide on the selling prices.

- charge bidder in j^{th} slot the $(j+1)^{th}$ highest bid

Note: when $k=1$, same as a Vickrey auction.

Fact: GSP auctions = dominant paradigm in sponsored search.

Question: but are they truthful?

Discussion

per-click

Example: $k = 2$. $\alpha_1 = .1$, $\alpha_2 = .05$. $v_1 = 10, v_2 = 9, v_3 = 6$.

- assume bidders #2 & #3 bid truthfully

Case 1 [$b_1 = 10$]: utility $= .1(10 - 9) = .1$

($.1$: probability of a click in slot #1; $(10-9)$: utility if there's a click)

Case 2 [$b_1 = 8$]: utility $= .05(10 - 6) = .2$

($.05$: α_2; 10: v_1; 6: new price per click)

GSP is not truthful!

Truthful alternative: Vickrey-Clarke-Groves (VCG) auction.

Question: Why not VCG?

1. didn't know about it
2. inertia
3. used to be easier to explain
4. short-term revenue loss
5. long-term revenue gain unclear